# Abstract


The search for optimal algorithms is at the core of computer science since its emergence as a field. Yet for the majority of the studied problems we do not know whether state-of-the-art algorithms are the best possible. Among the most popular problems in P are those that have standard algorithms that run in $O(n^c)$ time for an input of size $n$, where $c = 2$ or 3. For $c = 3$ (cubic time), we can find combinatorial matrix multiplication problems, and computing all-pairs-shortest-paths (APSP) of a directed real-weighted graph. For $c = 2$ (quadratic time), we can find many fundamental problems, such as 3SUM, and many basic matching problems between strings, curves, and point-sequences, such as Edit Distance, Longest Common Subsequence, Discrete Fréchet Distance, Geometric Edit Distance, and Dynamic Time Warping. These problems are usually referred to as "quadratic problems".

In this thesis, we present improved algorithms and decision trees for the following core problems.

- We improve the $(2k-2)$-linear decision tree bound for $k$-SUM and the subquadratic algorithm for the famous 3SUM problem, both obtained by Grønlund and Pettie [104]. Follow-up to our work, Kane, Lovett, and Moran [114] obtained the breakthrough of showing a $2k$-linear decision tree for $k$-SUM with near-linear depth. Chan [55] improved our 3SUM algorithm and currently holds the record for the algorithm with the best runtime bound for the problem.

- We give the first subquadratic algorithms for computing Dynamic Time Warping (DTW) and Geometric Edit Distance (GED) between two point-sequences in $\mathbb{R}$ (and also in $\mathbb{R}^d$, for any constant $d$, when the underlying metric is polyhedral), breaking the nearly 50 years old quadratic time barrier for these problems. The DTW measure is an extremely popular matching method, being massively used in dozens of applications.

  For computing the related Discrete Fréchet Distance, we show linear decision trees with near-linear depth, for any fixed dimension, when the underlying metric is polyhedral.

- We give improved strongly-polynomial subcubic algorithms for solving the high-dimensional (e.g., $\mathbb{R}^n$) Closest Pair problem under the $L_\infty$ metric, and give improved runtime analysis for computing the related *dominance product* matrix of $n$ points in dimension polynomial in $n$. Computing the dominance product itself is an important task, since it is applied in many algorithms, in addition to our Closest Pair algorithm, as a major black-box ingredient.

- Another result, unrelated to the main motif of the thesis, shows the existence of sparse (subquadratic) *diameter spanners* with various size-stretch trade-offs, and gives efficient algorithms to construct them. That is, given a directed graph $G = (V, E)$ and a stretch factor $t > 0$, a subgraph $H = (V, E_H \subseteq E)$ is a *t-diameter spanner* iff $\text{diam}(H) \leqslant \lceil t \cdot \text{diam}(G) \rceil$, where $\text{diam}(H)$ and $\text{diam}(G)$ denote the diameter of $H$ and $G$, respectively.